Developing Methods to Cross-validate Nonlinear Models of Synchronization

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Sequence Production Lab

Introduction

Computational models of synchronization

- Humans tend to anticipate the beat when they synchronize with sound: Anticipatory synchronization (Repp et al., 2011)
- Delay-coupled nonlinear dynamical systems can capture anticipatory synchronization (Stepp & Turvey, 2010; Voss, 2000)
- Coupling strength and time delay can model group synchronization (Demos et al., 2019)

Model validation

- **Overfitting problem**: models with more parameters produce better fits, but may not generalize well to new data (Hastie et al., 2017)
- **Cross-validation**: divide data into matched Train and Test sets; train model parameters on Train set; compare Train and Test fits (Hastie et al., 2017; James et al., 2013)
  - Use data from other participants (surrogates) to evaluate error in model fits to Test data

Method

Modelling

Delay-coupling model (nonlinear)

$\dot{\theta}_1 = \omega_{met}$ Driver oscillator (metronome cue)

$\dot{\theta}_2 = \omega + \kappa\,(\theta_1 - \theta_{2,\tau})$ Driven oscillator (tapper)

ω: Intrinsic frequency; κ: Coupling strength; τ: Time delay

Linear model

$\dot{\theta}_1 = \omega_{met}$ Driver oscillator (metronome cue)

$\dot{\theta}_2 = \omega$ Driven oscillator (tapper)

Experiment

Participants: 24 adults assigned to 12 pairs:
Musically untrained (6 pairs) and trained (6 pairs)

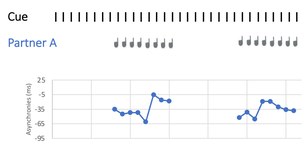
Spontaneous Production Rate (SPR) task

- Participant taps melody at a steady, uncued rate

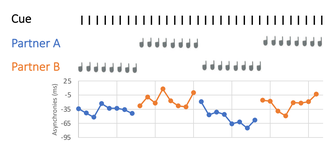
Synchronization tasks

- Participant taps melody in sync with auditory cue set at participant's own SPR or their partner's SPR)

Solo synchronization

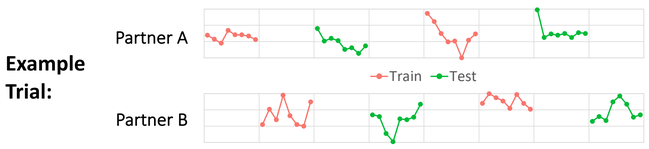


Joint (turn-taking) synchronization

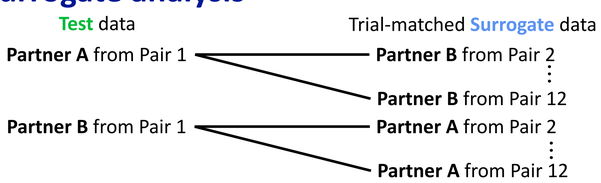


Cross-validation

Data divided into balanced Train and Test sets



Surrogate analysis



Results

Model parameters: Intrinsic frequency (SPR task)

Spontaneous production rates are consistent across trials and capture intrinsic frequency

Delay-coupling model (nonlinear)

$\dot{\theta}_1 = \omega_{met}$

$\dot{\theta}_2 = \omega + \kappa\,(\theta_1 - \theta_{2,\tau})$

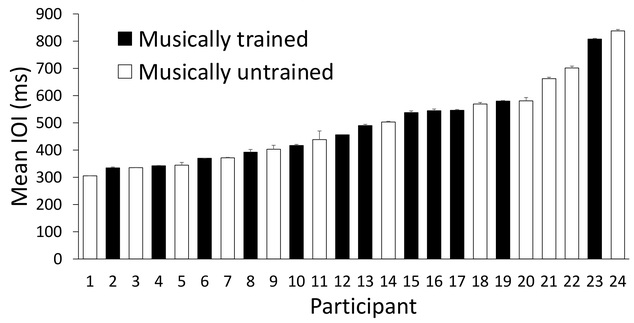
Intrinsic frequency; Coupling strength; Time delay

Linear model

$\dot{\theta}_1 = \omega_{met}$

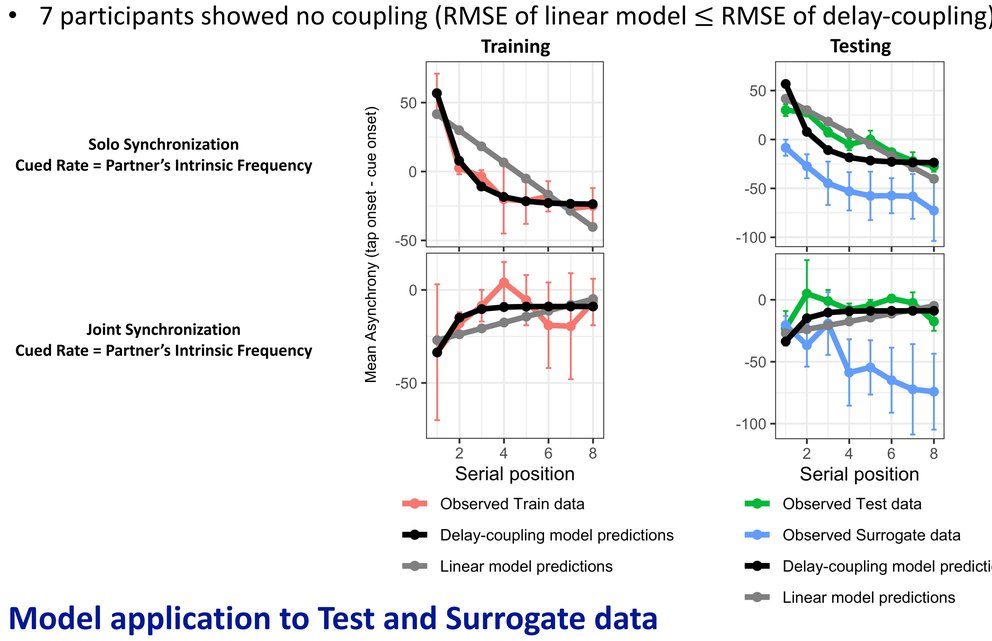
$\dot{\theta}_2 = \omega$

Intrinsic frequency



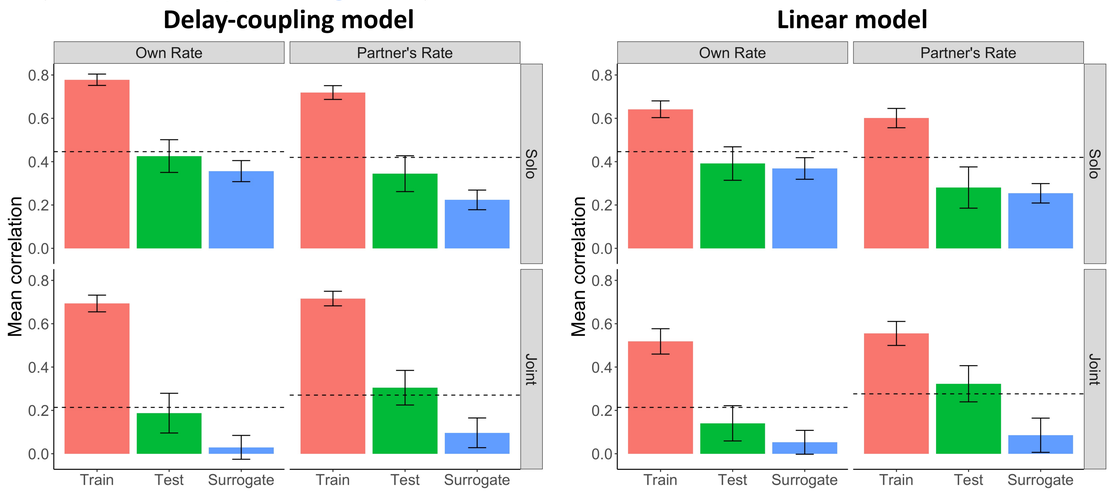
Model fits to Train data

- Optimal time delay ($\tau = 10.15$) was fixed across delay-coupling models and participants
- Root mean squared error (RMSE) used to select best model fits
- 7 participants showed no coupling (RMSE of linear model $\leq$ RMSE of delay-coupling), removed



Model application to Test and Surrogate data

Asynchronies predicted by model (fitted to Train data) correlated with observed asynchronies (from Train, Test, or Surrogate set)



Horizontal dashed line: correlation between observed Train data and observed Test data = Expected value that the model can reach for Test data

- Good fit for Test data, underfitting for Surrogate data

ANOVA (Rate x Condition x Model x Set) on z-transformed correlation coefficients (r)

- **Delay-coupling model**: Train > Test > Surrogate (p's < 0.03)
- **Linear model**: Train > Test = Surrogate ($p < .0001$, $p = .16$)
- Delay-coupling model > Linear model ($p < .001$)
- Solo > Joint ($p = .03$)

Discussion

Cross-validation of delay-coupling model

- Delay-coupling model can generalize: Fits to Test data significantly better than fits to Surrogate data

Delay-coupling vs linear model

- Linear model (without coupling) performed worse than nonlinear model (with coupling and time delay) in fitting to synchronization behavior
- Linear model can help detect when a participant is not coupled (linear model >= delay-coupling model)
- Linear model yielded similar correlations with Test and Surrogate data
  - Suggests linear model generalizes less well

Effect of social context on model fits

- Model's predicted asynchronies match observed data better for the Solo task than for the Joint task
  - Suggests that the social presence of partner may influence unaccounted model variance

Future directions

- Further investigate influences of social context
- Test effects of musical training on model fits

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Acknowledgments

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